Optimal control problem for the degenerate phase field system of equations

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Abstract:

In this work, the optimal control problem for the degenerate linear system of the phase field is considered

$$v(x,0)=\varphi(x), \quad x\in\Omega, \tag{1}$$

$$\theta\frac{\partial v}{\partial n}(x,t)+(1-\theta)v(x,t)=0, \quad (x,t)\in\partial\Omega\times(0,T), \tag{2}$$

$$\theta\frac{\partial w}{\partial n}(x,t)+(1-\theta)w(x,t)=0, \quad (x,t)\in\partial\Omega\times(0,T), \tag{3}$$

$$v_t(x,t)+lw_t(x,t)=k\Delta v(x,t)+u_1(x,t), \quad x\in\Omega\times(0,T), \tag{4}$$

$$\Delta w(x,t)+\alpha w(x,t)+\beta v(x,t)+u_2(x,t)=0, \quad x\in\Omega\times(0,T), \tag{5}$$

$$\|u_1\|^2_{L_2(0,T;L_2(\Omega))}+\|u_2\|^2_{L_2(0,T;L_2(\Omega))}\le R^2, \tag{6}$$

$$J(v,w)=\frac{1}{2}\|v-\tilde v\|^2_{L_2(0,T;L_2(\Omega))}+\frac{1}{2}\|w-\tilde w\|^2_{L_2(0,T;L_2(\Omega))}\to\inf, \tag{7}$$

where $\theta,l,k,\alpha,\beta,R$ are constants, $k>0$, $\tilde v,\tilde w\in L_2(0,T;L_2(\Omega))$ are given functions, v, w are unknown functions, $(u_1,\ u_2)$ is function of control. The goal is to minimize (7) when (u_1,u_2) satisfy condition (6). Here we use results on the existence of the degenerate initial value problem and the optimal control problem which presented in [1]. Simplified version of system (1)–(7) is considered in [2]. So, we are looking through the work in order to prove the conditional gradient numerical method convergence for the control problem. Besides, we've described the algorithm of numerical solution constructing.

Keywords: optimal control, degenerate evolution, numerical solution

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