Generalized and fundamental solutions of M. Biot equations for subsonic transport load

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Abstract: A special class of dynamic problems of continuum mechanics are transportation problems in which the acting external forces and loads moving at certain velocities and their shape does not change over time. By this type of system equations of motion of the medium depends strongly on the Mach numbers: the ratio of the motion velocity of the transport load to the sound velocities of propagation of disturbances in the medium , which may be several.

Here two-component medium of M. Biot consisting of solid and fluid components is considered under action of subsonic transport loads. This medium has three sound velocities [1]. It is assumed that the mass forces acting in the medium, move with c constant velocity along the axis z and represented as $G_i = G_i(x_1, x_2, z + ct)$ and motion equations of such medium are

$$(\lambda + \mu)u_{sj,ji} + Qu_{fj,ji} + \mu u_{si,jj} + G_{si} = c^2(\rho_{11}u_{si,33} + \rho_{12}u_{fi,33})$$
$$Qu_{si,ji} + Ru_{fj,ji} + G_{fi} = c^2(\rho_{12}u_{si,33} + \rho_{22}u_{fi,33}), \qquad (1)$$

where u_{si}, u_{fi} are the elastic and fluid components of the displacement vector, G_{si}, G_{fi} are the body forces acting respectively on the solid and fluid components, $u_{i,j} = \partial u_i/\partial x_j'$. The constants λ, μ, Q, R have the dimension of stress, $\rho_{11} = (1 - m)\rho_s - \rho_{12}$, $\rho_{22} = m\rho_f - \rho_{12}$, ρ_s and ρ_f are densities of the elastic and fluid components, m is the porosity of the medium.

By using the generalized Fourier transformation the fundamental and generalized solutions have been constructed for (1). The results of the numerical calculations, illustrating the Green tensor and pictures of vector fields displacement for solid and fluid components of the medium are presented.

Constructed solutions can be used to investigations the dynamics of the massif in the neighborhood of underground constructions such as tunnels, transport pipelines depending on the properties of water saturation of the medium, the velocity and type of existing transport loads.

Keywords: fundamental solutions, generalized solutions, elastic medium, Bio medium

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References

[1] M.A. Biot, "Mechanics of deformation and acoustic propagation in porous media", *Journal of Applied Physics*, Vol. 33, pp.1482-1498, 1962