About the defect indices of fourth-order differential operator with rapidly oscillating coefficients

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Abstract: We study in the space $L_2(0,\infty)$ of a minimum nonsemibounded differential operator L_0 [1], [2], generated by the differential expression

(1) $$ly = y^{(4)} - (h(x)y')' - q(x)y, \quad x \in (0;\infty)$$

(2) $$y^{(4)} - (h(x)y')' - q(x)y = \lambda y,$$

Where $\lambda \in \mathbf{C}$, $Im\,\lambda \neq 0$, the function $q(x)$ satisfies the conditions of regularity Titchmarsh-Levitan [3], and $h(x)$ is rapidly oscillating disturbance, which does not meet the conditions of the Titchmarsh-Levitan.

$$\varphi(\xi,\lambda) = \frac{h(x)}{4q^{\frac{1}{4}}(x)(q(x)+\lambda)^{\frac{1}{4}}}, \qquad \omega(\xi,\lambda) = \frac{q'(x)}{8q^{\frac{1}{4}}(x)(q(x)+\lambda)}, \qquad x = \nu(\xi).$$

(3) $$\left|\int_{\xi}^{\infty} \varphi(s,\lambda)ds\right| < \infty,$$

$$\varphi_1(\xi,\lambda) = \int_{\xi}^{\infty} \varphi(s,\lambda)ds.$$

(4) $$\mu_0\varphi_1(\xi,\lambda) \in L_1(0,\infty),$$

(5) $$\varphi_1(\xi,\lambda)\omega(\xi,\lambda) \in L_1(0,\infty),$$

(6) $$\mu_0\varphi_1^2(\xi,\lambda) \in L_1(0,\infty).$$

Theorem 0.1. *Let the*

a: $q(x) \geq c|x|^{\frac{4}{3}+\varepsilon}, \quad c > 0, \quad \varepsilon > 0, |x| \geq R;$
b: $q'(x)$, $q''(x)$ *do not change sign for sufficiently large* $R > 0$ *for* $|x| \geq R$*;*
c: $q'(x) = o\left(q^{\zeta}(x)\right)$ *when* $x \in (0;\infty)$*, where* $0 < \zeta < \frac{5}{4}$*,*

and the following conditions (3)*,* (4)*,* (5) *and* (6)*. Then the deficiency indices of the operator are equal (3.3).*

Keywords: Defect indices, differential operator, rapidly oscillating coefficients.

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