Hardy-type inequality for a fractional integral operator in q-analysis

Shaimardan, S

Faculty of Mathematics and Mechanics, L.N. Gumilyov Eurasian National University, Ástana, Kazakhstan
E-mail: email@address

Abstract: We consider the operator $I_{q,n}$ the following form

$$I_{q,n}f(x)=\frac{1}{\Gamma_q(n)}\int\limits_0^\infty \mathcal{X}_{(0,x]}(s)K_{n-1}(x,s)f(s)d_qs,$$

which is defined for all $x>0$ [1]. where $K_{n-1}(x,s)=(x-qs)_q^{n-1}$.

Then the q-analog of the two-weighted inequality for the operator $I_{q,n}$ of the form

$$\left(\int\limits_0^\infty u^r(x)\left(I_{q,n}f(x)\right)^r d_qx\right)^{\frac{1}{r}}\leq C\left(\int\limits_0^\infty v^p(x)f^p(x)d_qx\right)^{\frac{1}{p}}. \tag{1}$$

which has several applications in various fields of science. Where C a positive constants independent of f and $u(\cdot),v(\cdot)$ are positive real valued functions on $(0,\infty)$, i.e. weight functions.

Theorem. Let $1<r<p<\infty$. Then the inequality (1) holds if and only if $Q_{n-1}<\infty$ holds, where

$$\begin{aligned} Q_m^{n-1} &= \left\{\int\limits_0^\infty\left(\int\limits_0^\infty \mathcal{X}_{(0,z]}(s)K_m^{p'}(z,s)v^{-p'}(s)d_qs\right)^{\frac{p(r-1)}{p-r}}\right.\\ &\times\left(\int\limits_0^\infty \mathcal{X}_{[z,\infty)}(x)K_{n-m-1}^r(x,z)u^r(x)d_qx\right)^{\frac{p}{p-r}}\\ &\times\left. D_q\left(\int\limits_0^\infty \mathcal{X}_{(0,z]}(s)K_m^{p'}(z,s)v^{-p'}(s)d_qs\right)\right\}^{\frac{p-r}{pr}}. \end{aligned}$$

Moreover, $Q_{n-1}\approx C$, C is the best constant in (1).

Keywords: inequalities; Hardy-type inequalities, integral operator; q-calculus; q-integral

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References

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