Sobolev type equations of time-fractional order with periodical boundary conditions

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Abstract: The main purpose of the paper is studying of the unique solvability for initial boundary value problem to the fractional differential equation

$$D_t^\alpha Lx(t) = Mx(t) + N(t, x(t), x^{(1)}(t), \dots, x^{(m-1)}(t)), \tag{1}$$

where the operators L and M are differential operators with respect to the spatial variables, N is a nonlinear operator, $\alpha > 0$, $m \in \mathbb{N}$, $m - 1 < \alpha \leq m$, D_t^α is the Gerasimov–Caputo derivative. Initial conditions have the Cauchy form and boundary conditions are periodical with respect to every spatial variable on a parallelepiped. Such problems arise in many engineering and scientific disciplines as the mathematical modelling of systems and processes in the fields of physics, chemistry, aerodynamics, electrodynamics of complex medium, polymer rheology [1]. Using some results on fractional differential equations in Banach spaces from [2] the local unique solvability conditions are found for the initial boundary value problem to equation (1). General results are applied to the research of the time-fractional order Benjamin–Bona–Mahony–Burgers and Allair partial differential equations.

Equations of form (1) not solved with respect to the time derivative are called Sobolev type equations [3]. Some classes of Sobolev type fractional equations were studied in [4].

Keywords: Caputo fractional derivative, Sobolev type equation, nonlinear equation, initial boundary value problem, periodical boundary condition

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References

[1] F. Mainardi, G. Spada, Creep, relaxation and viscosity properties for basic fractional models in rheology, *The European Physics Journal, Special Topics*, **193**, 133-160 (2011).

[2] M.V. Plekhanova, Nonlinear equations with degenerate operator at fractional Caputo derivative, *Mathematical Methods in the Applied Sciences*, in press (2016). DOI: 10.1002/mma.3830

[3] G.A. Sviridyuk, V.E. Fedorov, *Linear Sobolev Type Equations and Degenerate Semigroups of Operators*, Utrecht, VSP, Boston, 2003.

[4] K. Balachandran, and S. Kiruthika, Existence of Solutions of Abstract Fractional Integrodifferential Equations of Sobolev Type, *Comput. Math. Appl.*, **64**, 3406-3413 (2012).