Asymptotics solutions of a singularly perturbed integro-differential problem with rapidly oscillating coefficients

Burkhan KALIMBETOV [1], Dinmukhambet SAPAKOV [1]

[1] *Department of Mathematics, H.A. Yasawi International Kazakh-Turkish University, Turkestan, Kazakhstan*
E-mail: bkalimbetov@mail.ru, sapakov@mail.ru

Abstract: In this paper we consider the Cauchy problem for a singularly perturbed integro-differential systems with rapidly oscillating coefficients:

$$\varepsilon\dot{z}-A(t)z-\varepsilon\varphi(t)cos\frac{2\beta(t)}{\varepsilon}Bz-\int\limits_{t_0}^{t}k(s)z(s,\varepsilon)ds=h(t),\ z(t_0,\varepsilon)=z^0,\ t\in[0,T], \tag{1}$$

where $z(t,\varepsilon)=\{u(t,\varepsilon),\vartheta(t,\varepsilon)\}$ is an unknown vector-function, $z^0=\{u^0,\vartheta^0\}$ is a known constant vector, $h(t)=\{h_1(t),h_2(t)\}$ are known vector-function, $A(t)$, $k(t)$, B are given matrices, $\varphi(t)$, $\beta(t)>0$ are known functions, $\varepsilon>0$ is a small parameter. It is required to construct the main term of asymptotics of a solution of (1) at $\varepsilon\to+0$.

A particular case of problem (1) (with $\beta(t)=\beta\cdot t, \beta=const, h(t)\equiv 0, k(t)\equiv 0$), describing the phenomenon of parametric amplification, been considered in [1–3]. We will illustrate the application of the regularization method [4, 5] for obtaining the principal term of the asymptotic behavior of the solution of problem (1).

Keywords: Integro-differential system, small parameter, asymptotic solution

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