Spectrum of Volterra integral operator of the second kind

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Abstract: In this paper we consider the singular Volterra integral equation with spectral parameter $\lambda \in \mathbb{C}$ of form

$$\varphi(t) - \lambda \int\limits_0^t K(t,\, \tau)\, \varphi(\tau)\, d\tau = f(t), \quad t > 0, \tag{1}$$

where

$$K(t,\, \tau) = K^{(1)}(t,\, \tau) + K^{(2)}(t,\, \tau), \tag{2}$$

$$K^{(1)}(t,\, \tau) = \frac{1}{2a\sqrt{\pi}} \cdot \frac{t^\omega + \tau^\omega}{(t-\tau)^{\frac{3}{2}}} \exp\left(-\frac{(t^\omega + \tau^\omega)^2}{4a^2(t-\tau)} \right), \tag{3}$$

$$K^{(2)}(t,\, \tau) = \frac{1}{2a\sqrt{\pi}} \cdot \frac{t^\omega - \tau^\omega}{(t-\tau)^{\frac{3}{2}}} \exp\left(-\frac{(t^\omega - \tau^\omega)^2}{4a^2(t-\tau)} \right),\ \omega\, 1/2. \tag{4}$$

We call such equations as the Volterra integral equations with 'incompressible' kernel [1]. It is shown that the corresponding homogeneous equation on $|\lambda| \geq \exp\{|\arg\lambda|\}, \quad \arg\lambda \in [-\pi, \pi]$ has a continuous spectrum, and the multiplicity of the characteristic numbers grows with increasing $|\lambda|$. We use the Carleman-Vekua regularization method. We introduce the characteristic integral equation. We prove that the initial integral equation has eigenfunctions, the multiplicity of which depends on the value of the spectral parameter λ. We prove the solvability theorem of the nonhomogeneous equation (1)–(4) in a case when the right-hand side of the equation belongs to a certain class.

Keywords: Volterra integral equation, spectrum, eigenfunction

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References

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