Spectral properties of a Laplace operator with Samarskii–Ionkin type boundary conditions in a disk

Makhmud SADYBEKOV [1], Nurgissa YESSIRKEGENOV [1,2]

[1] *Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan*
[2] *Department of Mathematics, Imperial College London, London, United Kingdom*
E-mail: sadybekov@math.kz, n.yessirkegenov15@imperial.ac.uk

Abstract: One of the most important problems in the theory of harmonic functions are Dirichlet and Neumann boundary value problems. In the one-dimensional case, or when considering the problem in a multidimensional parallelepiped, the main problems include also the periodic boundary value problems.

In this paper we consider spectral properties of a Laplace operator with boundary conditions of Samarskii-Ionkin type in a disk and prove the completeness of eigenfunctions.

Let $\Omega = \{z = (x, y) = x + iy \in C : |z| < 1\}$ be a unit disk, $r = |z|, \varphi = \arctan(y/x)$, $\Omega^+ = \Omega \cap \{y > 0\}$, and $\Omega^- = \Omega \cap \{y < 0\}$. We consider the spectral problem corresponding to the Laplace operator

$$-\Delta u(z) = \lambda u(z), |z| < 1$$

with local boundary conditions

$$u(1, \varphi) = 0, \ \ 0 \leq \varphi \leq \pi,$$

and with one of the nonlocal boundary conditions

$$\frac{\partial u}{\partial r}(1, \varphi) - \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0, \ \ 0 \leq \varphi \leq \pi,$$

or

$$\frac{\partial u}{\partial r}(1, \varphi) + \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0, \ \ 0 \leq \varphi \leq \pi.$$

We note that unlike the one-dimensional case the system of root functions of the problems consists only of eigenfunctions.

Keywords: Poisson equation, Samarskii–Ionkin type problem, eigenfunctions, eigenvalues

2010 Mathematics Subject Classification: 35J05, 35J25, 35P10