Stability of basis property of a type of problems with nonlocal perturbation of boundary conditions

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Abstract: This report is devoted to a spectral problem for a multiple differentiation operator with an integral perturbation of boundary conditions of one type which are regular, but not strongly regular:

$$l(u) \equiv -u''(x) = \lambda u(x), \ 0 < x < 1, \tag{1}$$

$$U_1(u) \equiv u'(0) - u'(1) + \alpha u(0) = \int_0^1 \overline{p(x)} u(x) dx, \ p(x) \in L_2(0,1), \tag{2}$$

$$U_2(u) \equiv u(0) - u(1) = 0. \tag{3}$$

Here $\alpha \neq 0$ is an arbitrary complex number.

The unperturbed problem ($p(x) \equiv 0$) has an asymptotically simple spectrum, and its system of normalized eigenfunctions creates the Riesz basis. We construct the characteristic determinant of the spectral problem with an integral perturbation of the boundary conditions. The perturbed problem can have any finite number of multiple eigenvalues. Therefore, its root subspaces consist of its eigen and (maybe) adjoint functions. It is shown that the Riesz basis property of a system of eigen and adjoint functions is stable with respect to integral perturbations of the boundary condition.

Throughout this note we mainly use techniques from our works [1].

Keywords: Riesz basis, regular boundary conditions, eigenvalues, root functions, spectral problem, integral perturbation of boundary condition, characteristic determinant

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References

[1] Sadybekov M.A., and Imanbaev N.S., “On the basis property of root functions of a periodic problem with an integral perturbation of the boundary condition”, *Differential Equations*. Vol.48, No.6, pp. 896–900, 2012.