Analytical solution of two-phase spherical Stefan problem by heat polynomials and integral error functions

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Abstract: On the base of the Holm model [1], we represent two phase spherical Stefan problem and its analytical solution, which can serve as a mathematical model for diverse thermo-physical phenomena in electrical contacts [2]. Spherical Stefan problem is reduced to linear problem with degenerate domain. Suggested solution is obtained from Integral Error Function and its properties which are represented in the form of series whose coefficients have to be determined:

$$u_1(x,t)=\sum_{n=0}^{\infty}A_{2n}(2a_1\sqrt{t})^{2n}\left[i^{2n}erfc\frac{-x}{2a_1\sqrt{t}}+i^{2n}erfc\frac{x}{2a_1\sqrt{t}}\right]$$
$$+\sum_{n=0}^{\infty}A_{2n+1}(2a_1\sqrt{t})^{2n+1}\left[i^{2n+1}erfc\frac{-x}{2a_1\sqrt{t}}-i^{2n+1}erfc\frac{x}{2a_1\sqrt{t}}\right], \qquad (1)$$

$$u_2(x,t)=\sum_{n=0}^{\infty}(2a_2\sqrt{t})^{n}\left[B_n i^{n}erfc\frac{-x}{2a_2\sqrt{t}}+C_n i^{n}erfc\frac{x}{2a_2\sqrt{t}}\right]. \qquad (2)$$

We find free interphase boundary, unknown coefficients A_{2n+1}, A_{2n}, B_n and C_n from boundary conditions utilizing Faa Di Bruno formula, Bell polynomials and Lebniz differentiation formula. To simplify calculations, we use Hermite polynomials to represent series (1) in the following form:

$$u_1(x,t)=\sum_{n=0}^{\infty}A_{2n}\sum_{m=0}^{n}x^{2n-2m}t^{m}B_{2n,m}+\sum_{n=0}^{\infty}A_{2n+1}\sum_{m=0}^{n}x^{2n-2m+1}t^{m}B_{2n+1,m}.$$

Convergence of solution series is proved.

Keywords: Stefan problem, integral error function, heat polynomials

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References

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