Weak solutions for nonlinear fractional differential equations with fractional separated boundary conditions in Banach spaces

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In this paper, we consider the following boundary value problem of fractional differential equation with fractional separated Boundary Conditions of the form

$$\begin{cases} {}^cD^r x(t) = f(x, x(t)), \quad t \in J = [0,\, 1],\ 1 < r \leq 2. \\ \alpha_1 x(0) + \beta_1({}^cD^p x(0)) = \gamma_1, \quad \alpha_2 x(1) + \beta_2({}^cD^q x(1)) = \gamma_2,\ 0 < p < 1 \end{cases} \tag{1}$$

where ${}^cD^r$ is the Caputo fractional derivative of order r, $f : J \times E \to E$ is a given function satisfying some assumptions that will be specified later, $\alpha_i,\ \beta_i,\ \gamma_i\ (i = 1,\, 2)$ are suitably chosen constants in E, with $\alpha_1 \neq 0$. and E is a Banach space with norm $\|.\|$.

The topic of fractional differential equations has been of great interest for many researchers in view of its theoretical development and widespread applications in various fields of science and engineering such as physics, biophysics, chemistry, statistics,economics, blood flow, phenomena, control theory, porous media, electromagnetic, and other fields. Boundary value problems with integral boundary conditions constitute an important class of problems and arise in the mathematical modeling of various phenomena such as heat conduction, wave propagation, gravitation, chemical engineering, underground water flow, thermoelasticity, and plasma physics. They include two-point, three-point, multipoint boundary value problems.

Our investigation relies upon the method associated with the technique of measures of noncompactness and the fixed point theorem of Mönch type.

Keywords and phrases: Caputo fractional derivative, Riemann Liouville integral, measure of noncompactness, fixed point, Banach space.

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