Investigation of Laplace transforms for Erlangen distribution of the first passage time into zero level of the semi-Markov random process with positive tendency and negative jumps

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Abstract: The investigations of the distributions for the processes of semi-Markov random process have an important value in the random process theory. The purpose of the present article is to find the Laplace transforms for Erlangen distribution of the semi-Markov random processes with positive tendency and negative jumps. Let's the sequence $\{\xi_k, \zeta_k\}_{k\geq 1}$ be given on the probability space $\{\Omega, \Im, P(\cdot)\}$, where the random variables ξ_k and ζ_k, $k \geq 1$ independent and identically distributed. Using these random variables we construct the following semi-markov random process:

$X(t) = z + t - \sum_{i=1}^{k-1} \zeta_i$, if $\sum_{i=1}^{k-1} \xi_i \leq t < \sum_{i=1}^{k} \xi_i$, $k = 1, 2, ...,$ $t, z \geq 0$.

$X(t)$ process is called as the semi-markov random process with positive tendency and negative jumps. Also let us denotethe random variable τ_z^0 the first falling time of the process into zero-level, as $\tau_z^0 = \min\{t : X(t) \leq 0\}$. We need to find the Laplace transform for distribution of random variable τ_z^0. Let's assume that the random variables ξ_1 and ζ_1 have an Erlangen distribution of third construction with the parameters λ and μ, respectively. According to total probability formula, we can write the following integral equation for the random variable τ_z^0 :

$L(\theta \mid z) = \frac{\lambda^3 e^{-\mu z}}{(\lambda+\mu+\theta)^3} + \frac{\lambda^3 \mu e^{-\mu z}}{2} \int_{s=0}^{\infty} s^2 e^{-(\lambda+\mu+\theta)s} \int_{\alpha=0}^{z+s} e^{\mu\alpha} L(\theta \mid \alpha) d\alpha ds.$

In this case, we get the following fourth order differential equation:

$$\begin{aligned} L^{(4)}(\theta \mid z) &- [3(\lambda+\theta)-\mu]L'''(\theta \mid z) + 3(\lambda+\theta)(\lambda+\theta-\mu)L''(\theta \mid z) \\ &- (\lambda+\theta)^2(\lambda+\theta-3\mu)L'(\theta \mid z) - \mu[(\lambda+\theta)^2 - \lambda^3]L(\theta \mid z) = 0. \end{aligned}$$

Then the general solution of this differential equation will be as follows:

$$L(\theta \mid z) = \frac{\lambda^3}{[\lambda+\theta-k_1(\theta)]^3} e^{k_1(\theta)z}.$$

This expression is the Laplace transform for relative distribution of random variable τ_z^0. Laplace transform for non-relative distribution of τ_z^0 will be

$$L(\theta) = \int_{z=0}^{\infty} L(\theta \mid z)\lambda^3 z^2 e^{-\lambda z} dz = \int_{z=0}^{\infty} c_1(\theta) e^{k_1(\theta)z} \lambda^3 z^2 e^{-\lambda z} dz = \frac{\lambda^3}{[\lambda - k_1(\theta)]^3} C_1(\theta).$$

Therefore, we will get the following characteristics for $\lambda m > \mu$:

$$\begin{aligned} E\tau_z^0 &= -L'(0) = \frac{3(\lambda+\mu)}{\lambda(\lambda-3\mu)}, \\ E(\tau_z^0 \mid z) &= \frac{3(1+z\mu)}{\lambda-3\mu}, \end{aligned}$$

and

$$D(\tau_z^0 \mid z) = \frac{3}{(\lambda-3\mu)^2} + \frac{12(3+z\lambda)\mu}{(\lambda-3\mu)^3}.$$

Keywords: Laplace transform, semi-Markov random process, random variable.

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