On asymptotic normality of solutions of the Cauchy problem of a parabolic equation with random right side

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Abstract: Consider the Cauchy problem for the heat equation

$$\frac{\partial u(t,x)}{\partial t} = \frac{1}{2}\frac{\partial^2 u(t,x)}{\partial x^2} + C\left(t,x\right), \quad u\left(0,x\right) = f\left(x\right), \tag{1}$$

where $t \geq 0,\ x \in \mathbb{R}, f\left(x\right),\ C(t,x)$are continuous bounded (generally speaking, random) functions. Then to solve the problem (1) we have

$$u\left(t,x\right) = M_x\left[f\left(W_t\right) + \int_0^t C\left(t-s, W_s\right)ds\right],$$

where W_s is Wiener process and sign M_x means taking the conditional expectation on all facing at the initial time t = 0 of point x, trajectories of W_t process. Next we consider the special case of the equation (1), namely, consider the equation

$$\frac{\partial u(t,x)}{\partial t} = \frac{1}{2}\frac{\partial^2 u(t,x)}{\partial x^2} + g(x)\dot{W}_t, u\left(0,x\right) = f\left(x\right), \tag{2}$$

where $t \geq 0,\ x \in \mathbb{R}, f\left(x\right), g(x)$ are limited continuous functions, $\dot{W}_t$ is "white noise" $[1,2]$.

Theorem 1. Let $f\left(x\right), g\left(x\right)$ be continuous and bounded functions. Then, if the function $g\left(x\right)$ at $|x| \to \infty$ satisfies relation $|g\left(x\right) - \sigma\left(x\right)| \to 0$, where $\sigma\left(x\right) = \sigma_1\ , x > 0;;\ \sigma\left(x\right) = \sigma_2\ ,\ x < 0$; then distribution of the random function $\frac{u(t,x)}{\sqrt{t}}$, where $u\left(t,x\right)$ is a solution of Cauchy problem given by equation (2), which at $t \to \infty$ converges to distribution of the normal random variable with parameters $\left(0, \frac{(\sigma_1+\sigma_2)^2}{4}\right)$.

Keywords: parabolic equation, Wiener process, white noise, asymptotic normality, infinitesimal operator

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References

[1] Venttsel, A.D., "The course of the theory of random processes", *M .: Nauka*, 1975.
[2] Akanbay, N., "Fundamentals of probability theory, mathematical statistics and the theory of random processes", *Almaty .: ed. "Kazakh University"*,2007.