On a problem of reconstruction of a discontinuous function by its Radon transform

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Abstract: A problem of reconstruction of a discontinuous function by its Radon transform is considered. Almost all of mathematical models of tomography include objects with discontinuous properties, but the vast majority of the algorithms applying for reconstruction of a searched-for function require its smoothness or at least continuity. Thus an employment of such algorithms for reconstruction of discontinuous functions leads to errors that increase the errors arising usually at the reconstruction of smooth functions in 3–10 times. Hence a development of the algorithms and numerical tools for reconstruction of discontinuous objects by tomographic data is an important task. An expression "breaks recovery" includes few problems [1–3]. First of them is the problem of visualization of a set of breaking points. This problem consists in obtaining the image, on which the set of breaking points of a function can be recognize easily. A result of the second problem (the identification of discontinuities) should be mathematical description of the points or lines of discontinuities. The third problem consists in determination of jump values.

The problem of reconstruction of a set of breaking points is connected closely with the problem of reconstruction of discontinuous functions. One of approaches to the numerical solution for the problem of reconstruction of discontinuous functions reconstruction consists in the next sequential steps: a) a visualization of a set of breaking points; b) an identification of this set; c) a determination of jump values; d) an eliminations of discontinuities. We consider three of listed problems except for the problem of jump values determining. The problems are investigated by mathematical modeling with usage of numerical experiments. The descriptions of simulations and their results are represented. The results of simulation are satisfactory and allow us to hope for the further development of the approach.

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Keywords: tomography, Radon transform, discontinuous function, visualization of breaking points, identification, elimination of breaks, simulation

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