Numerical solution of $2D$-vector tomography problem using the method of approximate inverse

Ivan SVETOV[1], Svetlana MALTSEVA[2], Anna POLYAKOVA[3]

[1,2,3] *Sobolev Institute of Mathematics, Russia*
[1,2,3] *Novosibirsk State University, Russia*
E-mail: [1]*svetovie@math.nsc.ru,* [2]*sv_maltseva@mail.ru,*
[3]*apolyakova@math.nsc.ru*

Abstract: The problem of vector tomography is considered in this work in the following formulation. Let a certain vector field $\mathbf{v}$ be given in a bounded domain of $\mathbb{R}^2$ filled by a medium in which the probing radiation propagates along straight lines. We have to reconstruct this field from its known longitudinal and (or) transverse ray transforms.

We propose an algorithm for solution of the vector tomography problem. The algorithm based on so called method of approximate inverse developed by A.K. Louis and his pupils [1–3]. The idea of the method of approximate inverse is as follows. Let $A : H \to K$ be a linear bounded operator. It is required to find an approximate solution (a function f) of the operator equation $Af = g$ for given $g \in K$. Mollifiers e^y_γ are used for solving. This functions have a properties $\langle e^y_\gamma, e^y_\gamma \rangle_H = 1$ and $\langle f, e^y_\gamma \rangle_H \approx f(y)$. Let A^* be an adjoint operator for A. Hence, equation $A^*\psi^y_\gamma = e^y_\gamma$ has a solution $\psi^y_\gamma \in K$ and

$$f(y) \approx \langle f, e^y_\gamma \rangle_H = \langle f, A^*\psi^y_\gamma \rangle_H = \langle Af, \psi^y_\gamma \rangle_K = \langle g, \psi^y_\gamma \rangle_K.$$

The authors were supported partially by the Russian Foundation for Basic Research (project 14-01-31491-mol_a)

Keywords: approximate inverse, vector tomography, ray transform, solenoidal vector field, potential vector field

2010 Mathematics Subject Classification: 44A12, 65R10, 65R32, 94A08

References

[1] Louis, A.K., Maass, P., "A mollifier method for linear operator equations of the first kind", *Inverse Problems*, Vol.6, pp. 427–440, 1990.

[2] Louis, A.K., "Approximate inverse for linear and some nonlinear problems", *Inverse Problems*, Vol.12, pp. 175–190, 1996.

[3] Rieder, A., Schuser, T., "The Approximate Inverse in Action with an Application to Computerized Tomography", *SIAM Journal on Numerical Analysis*, Vol.37, No.6, pp. 1909–1929, 2000.