Existence of eigenvalues of problem with shift for an equation of parabolic-hyperbolic type

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Abstract: In the paper a spectral problem for an operator of parabolic-hyperbolic type of I kind with non-classical boundary conditions is considered. The problem is considered in a standard domain. The parabolic part of the space is a rectangle. And the hyperbolic part of the space coincides with a characteristic triangle. We consider a problem with the local boundary condition in the domain of parabolicity and with the boundary condition with displacement in the domain of hyperbolicity.

Let $\Omega \in R^2$ be a finite domain bounded for $y > 0$ by the segments AA_0, A_0B_0, B_0B, $A = (0, 0)$, $A_0 = (0, 1)$, $B_0 = (1, 1)$, $B = (1, 0)$, and for $y < 0$ by the characteristics $AC: \ x + y = 0$ and $BC: \ x - y = 1$ of an equation of the mixed parabolic-hyperbolic type

$$Lu = \left\{ \begin{array}{c} u_x - u_{yy}, y > 0 \\ u_{xx} - u_{yy}, y < 0 \end{array} \right\} = f(x, y). \tag{1}$$

Problem S. *Find a solution to Eq.* (1) *satisfying boundary conditions*

$$u|_{AA_0 \cup A_0B_0} = 0, \tag{2}$$

$$\alpha u\left(\theta_0\left(t\right)\right) = \beta u\left(\theta_1\left(t\right)\right), \quad 0 \le t \le 1, \tag{3}$$

where $\theta_0\left(t\right) = \left(\frac{t}{2}, -\frac{t}{2}\right)$, $\theta_1\left(t\right) = \left(\frac{t+1}{2}, \frac{t-1}{2}\right)$.

We prove the strong solvability of the considered problem. The main aim of the paper is the research of spectral properties of the problem. The existence of eigenvalues of the problem is proved.

Keywords: spectral problem, equation of parabolic-hyperbolic type, boundary condition with shift

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