On some spectral inequalities for a nonlocal elliptic problem

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Abstract: Let $\Omega \subset R^2$ be a bounded domain, symmetric with respect to the origin and with a smooth boundary $\partial\Omega$. This symmetry means that alongside with a point (x_1, x_2) her "opposite" point $x* = (-x_1, -x_2)$ also belongs to the domain. Let us denote $\partial\Omega_+ = \partial\Omega \cap \{x_1 \geq 0\}$, $\partial\Omega_- = \partial\Omega \cap \{x_1 < 0\}$.

In this paper we consider the following problem:

$$-\Delta u\left(x\right) = f\left(x\right), x \in \Omega,$$

in the domain Ω, and satisfying the following boundary conditions

$$u\left(x\right) = -u\left(x*\right), \frac{\partial u\left(x\right)}{\partial n_x} = \frac{\partial u\left(x*\right)}{\partial n_x}, x \in \partial\Omega_+.$$

Here n_x is a derivative in the direction of an outer normal to $\partial\Omega$.

Investigated problem is an analogue of the classical periodic boundary value problems in the case of non-rectangular region. Note that the problem P in the case of a circle was first formulated and investigated in [1].

We prove self-adjointness of the problem and show a method of constructing eigenfunctions.

We obtain an analogue of the Rayleigh type inequality and some spectral inequality for the first eigenvalue of the nonlocal problem.

Keywords: Laplace operator, nonlocal problem, eigenvalue, eigenfunctions, Rayleigh type inequality

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References

[1] M. .A. Sadybekov, and B. K. Turmetov, *Differential Equations.* **50**:2, 268–273 (2014).