Duality property of the noncommutative ℓ_∞ and ℓ_1 valued symmetric Hardy spaces

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Abstract: In this paper we consider the noncommutative $H_E(\mathcal{A};\ell_\infty)$ and $H_E(\mathcal{A};\ell_1)$ spaces and obtain some result on duality for these spaces, i.e we obtain the following result:

Theorem. Let E be an r-convex symmetric Banach function space on $[0;1]$ for some $0 < r < \infty$ and E do not contain c_0 or separable. Then

$$(i)\ \ (H_E(\mathcal{A};\ell_1))^* = L_{E^\times}(\mathcal{M};\ell_\infty)/J(H^0_{E^\times}(\mathcal{A};\ell_\infty))$$

isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_E(\mathcal{A};\ell_1)$ and $y \in H_{E^\times}(\mathcal{A};\ell_\infty)$, where $J(H^0_{E^\times}(\mathcal{A};\ell_\infty)) = \{x^* : x \in H^0_{E^\times}(\mathcal{A};\ell_\infty)\}$.

$$(ii)\ \ (L_E(\mathcal{M};\ell_1)/J(H^0_p(\mathcal{A};\ell_1)))^* = H_{E^\times}(\mathcal{A};\ell_\infty)$$

isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_E(\mathcal{A};\ell_1)$ and $y \in H_{E^\times}(\mathcal{A};\ell_\infty)$, where $J(H^0_{E^\times}(\mathcal{A};\ell_1)) = \{x^* : x \in H^0_{E^\times}(\mathcal{A};\ell_1)\}$.

This theorem is analogue of Theorem 5 in [1].

Keywords: von Neumann algebra, subdiagonal algebras, noncommutative vector valued symmetric Hardy spaces, duality.

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References

[1] T. N. Bekjan, K. Tulenov, and D. Dauitbek, The Noncommutative $H_p^{(r,s)}(\mathcal{A};l_\infty)$ and $H_p(\mathcal{A};l_1)$ spaces. *Positivity*, **19** 877–891 (2015).