On a class of fractional elliptic problems with an involution perturbation

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Abstract: To describe the problems, let $\Omega = \{(x,y) \in R^2 : 0 < x < 1, -\pi < y < \pi\}$. The paper is concerned with four boundary value problems concerning the fractional analogue of Helmholtz equation with a perturbation term of involution type in the space variable:

(1) $\quad D_x^{2\alpha}u(x,y) + u_{yy}(x,y) - \varepsilon u_{yy}(x,-y) - c^2u(x,y) = 0, (x,y) \in \Omega,$

where c, ε are real numbers, $D_x^{2\alpha}$ means $D_x^{2\alpha} = D_x^{\alpha}D_x^{\alpha}$ and the operator D_x^{α} acts by the variable x.
Here

$$D_x^{\alpha}u(x,y) = \frac{1}{\Gamma(1-\alpha)}\int_0^x (x-s)^{-\alpha}\frac{\partial u}{\partial s}(s,y)ds$$

is a Caputo differentiation operator of the $\alpha \in (0,1]$ order [1].

Regular solution of Equation (1) is a function $u \in C\left(\bar{\Omega}\right)$, such that $D_x^{\alpha}u, D_x^{2\alpha}u, u_{yy} \in C(\Omega)$. Since for $\alpha = 1 : {}_CD^1 = \frac{d}{dy}$, then $D_x^2 + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \Delta$. Therefore, Equation (1) is a nonlocal generalization of the Helmholtz equation, which at $\varepsilon = 0$ coincides with the Helmholtz equation.

We obtain for them existence and uniqueness results based on the Fourier method.

Keywords: Caputo operator, Helmholtz equation, involution, fractional differential equation, Mittag-Leffer function, boundary value problem

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