On solvability of some boundary value problems for a biharmonic equation with periodic conditions

V.KARACHIK [1], S.MASANOV [2],B.Kh.TURMETOV [3]

[1] *Department of Mathematics,South Ural State University, Chelyabinsk, Russia Fatih University,*
E-mail: valerykarachik@mail.ru

[2] *Department of Mathematics, Akhmet Yasawi University, Turkestan, Kazakhstan*
E-mail: meeir0808@gmail.com

[3] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*
E-mail: turmetovbh@mail.ru

Abstract:

Let $\Omega = \{x \in R^n : |x| < 1\}$ be a unit ball, $n \geq 2, \partial\Omega$ be a unit sphere. For any point we compare its "opposite" point $x^* = (-x_1, ..., -x_n) \in \Omega$, and denote $\partial\Omega_+ = \partial\Omega \cap \{x \in R^n : x_n \geq 0\}$, $\partial\Omega_- = \partial\Omega \cap \{x \in R^n : x_n \leq 0\}$, $I = \partial\Omega \cap \{x \in R^n : x_n = 0\}$.

Consider the following problem in the domain Ω:

$$\Delta^2 u(x) = f(x), x \in \Omega \tag{1}$$

$$D_\nu^m u(x) = g(x), x \in \partial\Omega \tag{2}$$

$$D_\nu^{\ell_1} u(x) - (-1)^k D_\nu^{\ell_1} u(x^*) = g_1(x), x \in \partial\Omega_+ \tag{3}$$

$$D_\nu^{\ell_2} u(x) + (-1)^k D_\nu^{\ell_2} u(x^*) = g_2(x), x \in \partial\Omega_+ \tag{4}$$

where $D_\nu^m = \frac{\partial^m}{\partial \nu^m}, m \geq 1$, $\nu-$ is a normal vector, $k = 1, 2,\ 1 \leq m \leq 3, 1 \leq \ell_1 < \ell_2 \leq 3, \ell_j \neq m, j = 1, 2.$

The exact conditions for solvability of the problems are found. Note, that analogous problems for Poisson equations were studied in [?] - [?].

Keywords: biharmonic equation, periodic boundary value problem, solvability, existence and uniqueness of solution

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References

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