The gyroscope movement with variable moments of inertia

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Abstract: It took more than two hundred years since the publication of the equations of rigid body dynamics with a fixed point, but research is still ongoing. Great interest in this problem is caused by the fact that in the motion of rigid body with a fixed point are observed gyroscopic effects, widely used in modern technology. The topicality of considering the problem about motion of rigid body with a fixed point is also due to the need to allow for perturbing gravitational, electrical, magnetic and other forces, variability of inertia moment of the object and a wide application in practice.

This kind of problems can be reduced to the study of systems of nonlinear and with variable coefficients differential equations, generating analytical solutions of which offers great mathematical difficulty, and it is possible in a relatively small number of cases. Therefore, the construction of analytical solutions for a wide class of such problems is topical.

This paper discusses motion equations of an axisymmetric rigid body with a fixed point with variable moment of inertia, which is described by a system of nonlinear dynamic Euler equations

$$\begin{cases} \dfrac{d[A(t)]p}{dt} + [C(t) - B(t)]qr = M_x, \\ \dfrac{d[B(t)]q}{dt} + [A(t) - C(t)]rp = M_y, \\ \dfrac{d[C(t)]r}{dt} + [B(t) - A(t)]pq = M_z, \end{cases} \tag{1}$$

where $A(t)$, $B(t)$, $C(t)$ are body's inertia moments relative to x, y, z axes connected with the body; p, q, r are projections of the body's angular velocity vector on these axes; M_x, M_y, M_z are moments of external resistance forces.

The system of differential equations (1) is considered jointly with initial conditions

$$\begin{gathered} t = 0 : p(0) = p_0, \quad q(0) = q_0, \quad r(0) = r_0, \\ \dot{p}(0) = 0, \quad \dot{q}(0) = 0, \quad \dot{r}(0) = 0. \end{gathered} \tag{2}$$

The solution of this problem is obtained by the technique of partial discretization of non-linear differential equations, built by A.N. Tyurekhodzhayev on the base of the generalized functions theory.

References

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