On a problem for wave equation with data on the whole boundary

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Abstract: In this paper we propose a new formulation of boundary value problem for a one-dimensional wave equation in a rectangular domain in which boundary conditions are given on the whole boundary.

Let $\Omega \subset R^2$ be a rectangular domain, bounded by following lines: $AB : 0 \leq x \leq \ell$, $t = 0$, $BC : x = \ell$, $0 \leq t \leq T$, $CD : 0 \leq x \leq \ell, t = T$ and $AD : x = 0$, $0 \leq t \leq T$. Let $E = (T, T), F = (\ell - T, T)$ be points on a boundary CD.

We consider a nonhomogeneous wave equation in Ω

$$u_{tt} - u_{xx} = f(x, t). \tag{1}$$

It is well known that the Dirichlet problem for the wave equation (1) in a rectangular domain is ill-posed.

Problem. *Find a solution of Eq.(1), satisfying the boundary condition*

$$u|_{AB \cup BC \cup AD} = 0,$$

and conditions on the boundary CD:

$$u_t|_{DE} = 0,$$
$$\alpha u_x + \beta u_t|_{EF} = 0,$$
$$u|_{CF} = 0,$$

where α and β are real numbers.

We prove the well-posedness of boundary value problem in the classical and generalized senses, depending on the coefficients α and β. The proof of the well-posedness of the formulated problem is reduced to question of the existence and uniqueness of solutions of the corresponding functional equations.

Keywords: Wave equation, well-posedness of problems, classical solution, strong solution

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