Heisenberg's inequality in Morrey spaces

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Abstract: The (classical) Heisenberg inequality is given by

$$\|f\|_{L^2}^2 \lesssim \left(\int_{\mathbb{R}} (x-a)^2 |f(x)|^2 \, dx\right)^{\frac{1}{2}} \left(\int_{\mathbb{R}} (\xi-\alpha)^2 |\hat{f}(\xi)|^2 \, d\xi\right)^{\frac{1}{2}}, \tag{1}$$

where $f \in L^2(\mathbb{R})$, $a, \alpha \in \mathbb{R}$, and $\hat{f}$ denotes the Fourier transform of f. There are several generalizations of (1) in Lebesgue spaces over $\mathbb{R}^n$ (see [1–4]). In this talk we discuss a generalization of the Heisenberg inequality in Morrey spaces. We combine some Hardy type inequality and an interpolation inequality for the fractional power of the Laplacian to prove our result and to remove some restrictions in [2]. Our proof follows the idea in [3]. This is a joint work with Hendra Gunawan (Bandung Institute of Technology), Eiichi Nakai (Ibaraki University), and Yoshihiro Sawano (Tokyo Metropolitan University) (see [5]).

Keywords: Imaginary power of Laplace operators, fractional power of Laplace operators, interpolation inequality, Hardy's inequality, Heisenberg's inequality, Morrey spaces.

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