Characteristic problems for a loaded hyperbolic equation with the wave operator in the principal part

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Abstract: In this paper we consider an equation of the form

$$u_{xx} - u_{yy} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = = \lambda \sum_{i=1}^{n} a_i(x,y) D_{0\xi}^{\alpha_i} u \left(\frac{\xi + \alpha(\xi)}{2}, \frac{\alpha(\xi) - \xi}{2} \right), \tag{1}$$

where $\xi = x - y$, λ is the real number, $\alpha_1 < \alpha_2 < ... < \alpha_n = \alpha < 1$, D_{ax}^l is the operator of fractional integration as $l < 0$ and fractional differentiation as $l > 0$ of order $|l|$ with the starting point a and the end point x, $\alpha(\xi)$ is a C^2-diffeomorphism, and moreover

$$0 \leq \alpha(\xi) \leq 1, \xi \in [0,1], \alpha(0) = 0, \tag{2}$$

$\varphi, \psi \in C(\bar{J}), \varphi(0) = \psi(0)$,

$$a, b, c \in C^1(\Omega), a_i \in C(\bar{\Omega}), i = \overline{1,n}, \tag{3}$$

where $\bar{J}$ is the closure of the interval $J = \{(x,y), 0 < x < 1, y = 0\}$.

Assume $\Omega = \{(x,y) : 0 < x + y < 1, 0 < x - y < 1\}$ is the rectangular domain bounded by characteristic equation (1).

Problem G. In the domain Ω find the solution to equation (1) belonging to $C(\bar{\Omega}) \cap C^2(\Omega)$, satisfying the boundary conditions

$$u\left(\frac{x}{2}, \frac{x}{2}\right) = \psi(x), \qquad 0 \leq x \leq 1, \tag{4}$$

$$u\left(\frac{x}{2}, -\frac{x}{2}\right) = \varphi(x), \qquad 0 \leq x \leq 1, \tag{5}$$

where $\bar{\Omega}$ is the closure of the domain Ω.

Theorem. *Under the assumptions (2) and (3), the Goursat problem (1), (4) and (5) is always uniquely solvable.*

Keywords: hyperbolic equation, fractional derivative, Goursat problem, wave equation, loaded equation

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