A generalized Banach contraction principle on cone pentagonal metric spaces over Banach algebras

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Abstract: In this paper, we introduce the concept of cone pentagonal metric spaces over Banach algebras as a generalization of metric space and many of its generalization such as; cone metric space [1, 2], cone rectangular metric space [3], and cone pentagonal metric space [4]. Furthermore, we prove a generalized Banach contraction principle in such a space as follows:

Theorem 0.1. *Let* $(\mathcal{X}, d)$ *be a complete cone pentagonal metric space over Banach algebra* $\mathcal{B}$ *and* S *be a non normal solid cone in* $\mathcal{B}$*. Suppose* $T : \mathcal{X} \to \mathcal{X}$ *is a mapping that satisfies the following condition:*

$$d(Tx, Ty) \preccurlyeq kd(x, y) \text{ for all } x, y \in \mathcal{X},$$

where $k \in S$ *is a generalized Lipschitz constant such that the spectral radius* $\delta(k) < 1$*. Then* T *has a unique fixed point* x^* *in* $\mathcal{X}$*. Moreover, for any* $x \in \mathcal{X}$*, the iterative sequence* $\{T^i x\}$ $(i \in \mathbb{N})$ *converges to* x^**.*

Keywords: cone pentagonal metric spaces, Banach algebras, c-sequence, contraction mapping principle, fixed point.

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References

[1] L.G. Huang, X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, Journal of Mathematical Analysis and Applications, vol. 332, no. 2, 1468–1476, 2007.

[2] H. Liu, S. Xu, Cone metric spaces with Banach algebras and fixed point theorems of generalized Lipschitz mappings, Fixed Point Theory and Applications, vol. 2013, no. 320, 1–10, 2013.

[3] A. Azam, M. Arshad, I. Beg, Banach contraction principle on cone rectangular metric spaces, Applicable Analysis and Discrete Mathematics, vol. 3, no. 2, 236–241, 2009.

[4] M. Garg, S. Agarwal, Banach contraction principle on cone pentagonal metric space, Journal of Advanced Studies in Topology, vol. 3, no. 1, 12–18, 2012.