Six Point Implicit Methods for the Pure Second Derivatives of the Solution of First Type Boundary Value Problem for One Dimensional Heat Equation

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Abstract: We construct six point implicit difference boundary value problem for the first derivative of the solution $u(x,t)$ of the first type boundary value problem for one dimensional heat equation with respect to the time variable t. Furthermore, for the second order pure derivatives of $u(x,t)$ special six point implicit difference boundary value problems are proposed. A uniform approximation of the order $O(h^2+\tau^2)$(second order accurate in the spatial varıable x and second order accurate in time t) where h is the step size in spatial variable x and τ is the step size in time is achieved. It is assumed that the initial function belongs to the Hölder space $C^{10+\alpha}$, $0<\alpha<1$, the heat source function is from the Hölder space $C_{x,t}^{8+\alpha,4+\frac{\alpha}{2}}$, the boundary functions are from $C^{5+\frac{\alpha}{2}}$, and between the initial and the boundary functions the conjugation conditions of orders $q=0,1,2,3,4,5$ are satisfied. Theoretical results are justified by numerical examples.