About convergence of difference approximations for optimization problems described by elliptic equations with mixed derivatives and unbounded non-linearity

Aigul Manapova[1], Fedor Lubyshev[2]

[1] *Bashkir State University, Ufa, Republic of Bashkortostan, Russia*

aygulrm@yahoo.com

[2] *Bashkir State University, Ufa, Republic of Bashkortostan, Russia*

Abstract: This report is devoted to an optimal control problem for non-linear elliptic equation with mixed derivatives and unbounded non-linearity, namely, minimizing of a functional $J(g) = \int\limits_{\Omega} |u(x; g) - u_0(x)|^2 d\Omega$, on solutions $u(g)$ to problem

$$-\sum_{\alpha=1}^{2} k_{\alpha\alpha}(x)\frac{\partial^2 u}{\partial x_\alpha^2} - 2k_{12}(x)\frac{\partial^2 u}{\partial x_1 \partial x_2} + q(u)u = f(u),\ x \in \Omega;$$
$$u(x) = 0, \qquad x \in \partial\Omega = \Gamma,$$

obeying all admissible controls

$$g = \big(k_{11}, k_{22}, k_{12}\big) \in U = \Bigg\{k_{\alpha\beta} \equiv g_{\alpha\beta} \in W_\infty^1(\Omega), \alpha, \beta = 1, 2 :$$
$$\nu \sum_{\alpha=1}^{2} \xi_\alpha^2 \le \sum_{\alpha,\beta=1}^{2} k_{\alpha\beta}(x)\xi_\alpha\xi_\beta \le \mu \sum_{\alpha=1}^{2} \xi_\alpha^2,\ k_{\alpha\beta}(x) = k_{\beta\alpha}(x),$$
$$\forall x \in \Omega, \xi \ne 0, \xi \in \mathbb{R}^2, \left|\frac{\partial k_{\alpha\beta}}{\partial x_1}\right| \le R_1, \left|\frac{\partial k_{\alpha\beta}}{\partial x_2}\right| \le R_2, \alpha, \beta = 1, 2\Bigg\},$$

where $u_0^{(1)} \in W_2^1(\Omega_1)$ is a given function, $q(\eta)$, $f(\eta)$ are given functions of η. Suppose that the conditions imposed on the coefficients of the state equation are satisfied only in the vicinity of the exact solution values that indicates the presence of non-linearities of the unbounded growth.

Present work is dedicated to the development and study of difference approximations for the given class of optimization problems, namely, study of issues, such as the convergence of approximations with respect to the state, functional, and control (see, e.g., [1], [2] and the references therein).

Keywords: Elliptic equations with mixed derivatives, optimal control, objective functional, unbounded non-linearity, difference method

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