A third order of accuracy difference scheme for Bitsadze-Samarskii type multi-point overdetermined elliptic problem

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Abstract: Let us $k_i, \lambda_i, 1 \leq i \leq q; \lambda_0, T, \alpha$ are known nonnegative real numbers such that

$$(1) \qquad \begin{array}{l} \sum\limits_{i=1}^{q} k_i = 1, k_i \geq 0, i = 1, \ldots, q, \\ 0 < \lambda_1 < \lambda_2 < \ldots < \lambda_q < T, \lambda_0 \in (0, T), 0 < \alpha < 1. \end{array}$$

Consider the following multipoint multidimensional elliptic problem with overdetermination

$$(2) \qquad \begin{cases} -v_{tt}(t,x) - \sum\limits_{r=1}^{n} (a_r(x) v_{x_r}(t,x))_{x_r} + \sigma v(x) = g(t,x) + p(x), \\ x = (x_1, ... x_n) \in \Omega, 0 < t < T, \\ v(0,x) = \phi(x), v(T,x) - \sum\limits_{i=1}^{q} k_i v(\lambda_i, x) = \eta(x), \\ v(\lambda_0, x) = \zeta(x), x \in \overline{\Omega},\ v(t,x) = 0, x \in S, \end{cases}$$

where $\Omega = (0, \ell)^n$ is the open cube in R_n with boundary S, $\overline{\Omega} = \Omega \cup S$ and nonnegative real numbers σ, λ_0, λ_i, k_i, $1 \leq i \leq q$ are known, smooth functions a_r, ϕ, η, ζ, and f are given on Ω, $a_r(x) > 0$, $\forall x \in \Omega$. In [1], well posedness of problem 2 was established. A first and a second order difference schemes for its approximately solution were constructed. In [2], overdetermined problem for the multi-dimensional elliptic equation with Neumann boundary condition was investigated. The papers [3–5] are devoted to high order approximations of overdetermined elliptic problems without nonlocal boundary conditions.

In the present work, we construct a third order of accuracy difference scheme for problem 2 and establish stability, almost coercive stability and coercive stability estimates for its solution.

Keywords: nonlocal boundary condition,inverse elliptic problem, well-posedness, stability, coercive stability, overdetermination.

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