Stationary boundary-value problems of the anisotropic elastic medium dynamics

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Abstract: The solution of boundary value problems of dynamics of elastic media is one of the topical problems of mathematical physics, because by them is described a wide class of various geophysical processes, dynamic processes in materials and constructions that are widely used in bilding and engineering. The most are developed the mathematical methods for investigation of boundary value problems of the dynamics of isotropic elastic media. Anisotropic media have been little studied, which is connected with the complexity of constructing solutions of equations that are usually solved on the basis of numerical difference or finite element methods.

Here we consider the solution of problems of stationary oscillations with a frequency ω for anisotropic elastic media, which for complex amplitudes is described by a system of equations of the form

$$L_{ij}\left(\partial_x, -i\omega\right) u_j(x) + G_i(x) = 0, \tag{1}$$

$$L_{ij}\left(\partial_x, -i\omega\right) = C_{ij}^{ml}\partial_m\partial_l + \delta_{ij}\rho\omega^2, i, j, m, l = \overline{1, N} \tag{2}$$

Here $u_{i,j} = \partial u_i/\partial x_j$, δ_{ij} is Kronecker symbol, ρ is the density of the medium, G_i are the components of the mass force, C_{ij}^{ml} is the matrix of elastic constants, which has symmetry properties with respect to permutation of the indexes $C_{ij}^{ml} = C_{ij}^{lm} = C_{ji}^{ml} = C_{ml}^{ij}$ (everywhere the summation over repeated indices in the indicated range is assumed).

Two boundary value problems for complex amplitudes of displacements of the medium, which belong to the class of boundary value problems for elliptic systems of equations, are given. To solve them we use the method of generalized functions that allows the initial boundary value problem to lead to the solution of a system of differential equations in the space of generalized functions with a certain right-hand side from the class of singular generalized functions of the type of simple and double layers and to use the properties of fundamental solutions of such systems for constructing its generalized solutions.Considered the fundamental solutions, their symmetry properties and asymptotics. A generalization of the Somilyana formula for displacements is obtained which allows by known displacements and stresses on the boundary of body one to find the displacement amplitudes inside it. The analogue of Gauss's formula is presented, which determines the characteristic function of the domain of definition through surface integrals of the stress tensor. On

its basis singular boundary integral equations for solving the boundary value problems are constructed.

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