Well-posedness and energy decay of solutions to a Lamé system under boundary fractional derivative controls

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Abstract: In this paper, we investigate the existence and decay properties of solutions for the initial boundary value problem of the Lamé system with boundary control conditions of fractional derivative type:

$$(P) \qquad \begin{cases} u_{tt} - \mu\Delta u - (\mu+\lambda)\nabla(div\ u) = 0 \quad \text{in } \Omega \times (0,+\infty) \\ u = 0 \quad \text{in } \Gamma_0 \times (0,+\infty) \\ \mu\frac{\partial u}{\partial \nu} + (\mu+\lambda)(div\ u)\nu = -\gamma \partial_t^{\alpha,\eta} u \quad \text{in } \Gamma_1 \times (0,+\infty) \end{cases}$$

where $u = (u_1, u_2, ..., u_n)^T$, ν stands for the unit normal vector of Ω pointing towards the exterior and μ, λ, γ are positive constants. The notation $\partial_t^{\alpha,\eta}$ stands for the generalized Caputo's fractional derivative of order α with respect to the time variable. It is defined as follows

$$\partial_t^{\alpha,\eta} w(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} e^{-\eta(t-s)} \frac{dw}{ds}(s)\, ds \qquad \eta \geq 0.$$

The system is finally completed with initial conditions

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x).$$

Our purpose in this paper is to give a global solvability in Sobolev spaces and energy decay estimates of the solutions to the problem (P).

Keywords: Lamé system, Fractional feedback, Polynomial stability, Semigroup theory.

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