On a first-order partial differential equation

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Abstract: In this report we study a linear differential equation with first-order partial derivatives, where the coefficients of the equation are given on an unbounded set and has continuous first-order partial derivatives.

$$\frac{\partial u}{\partial x}+\left(\sum_{k=1}^{n} p_{1k}(x)y_k+g_1(x,y_1,\dots,y_n)\right)\frac{\partial u}{\partial y_1}$$

$$\dots+\left(\sum_{k=1}^{n} p_{nk}(x)y_k+g_n(x,y_1,\dots,y_n)\right)\frac{\partial u}{\partial y_n}=0 \qquad (1)$$

where $x \in I \equiv [x_0 \le x < +\infty),\ x_0>0,\ -\infty<y_1,\dots,y_n<+\infty,$

Theorem 1. If the following conditions are fullfilled:

(1) $p_{ik}(x), i=1,\dots,n,\ k=1,\dots,n$; are continuous differentiable on I.

(2) $p_{k-1,k-1}(x)-p_{kk}(x)\ge \alpha\varphi(x),\ x\in I,\ k=2,\dots,n.\ \alpha>0,\ \varphi(x)\in \mathbb{C}(I)$, $\varphi(x)>0,\ q(x)=\int\limits_{x_0}^{x}\varphi(s)ds\uparrow+\infty;$

(3) $\lim\limits_{x\to+\infty}\frac{|p_{ik}(x)|}{\varphi(x)}=0,\ i\ne k,\ i=1,2,\dots,n,\ k=1,2,\dots,n;$

(4) $\lim\limits_{x\to+\infty}\frac{1}{q(x)}\int\limits_{x_0}^{x}p_{kk}(s)ds=\beta_k,\ k=1,2,\dots,n,\ \beta_1<0;$

(5) $g(x,y) = colon\,(g_1(x,y_1,\dots,y_n),\dots,g_n(x,y_1,\dots,y_n))$ has a continuous partial derivatives on the set $x_0\le x<+\infty,\ x_0>0,\ -\infty< y_1,\dots,y_n<+\infty$, è $g_i(x,0,\dots,0)=0,\ i=1,\dots,n.$ $\|g(x,y)\|\le\delta(x)\|y\|$, $\lim\limits_{x\to+\infty}\frac{\delta(x)}{\varphi(x)}=0$, then the equation (1) has a integral basis, which attempts to zero at $x_0\to+\infty$.

Keywords: equation, first order partial derivatives

2010 Mathematics Subject Classification: 35B35

References

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