On the numerical solution of advection-diffusion problems with singular source terms

Ezgi Soykan[1], Mehmet Ahlatcioglu[1],Maksat Ashyraliyev[2]

[1] *Department of Mathematics, Yildiz Technical University, Turkey*

[2] *Department of Software Engineering, Bahcesehir University, Turkey*
maksat.ashyralyyev@eng.bau.edu.tr

Abstract: Partial differential equations with singular source terms are widely used in mathematical modeling of real-life systems in many different fields of science. Singular means that within the spatial domain the source is defined by a Dirac delta function. Solutions of the problems having singular source terms have lack of smoothness, which is generally an obstacle for standard numerical techniques [1–3].

In this work, we consider the initial-boundary value problem with singular source terms

$$(1) \quad \begin{cases} u_t + au_x = Du_{xx} + k_1\delta(x-\xi_1) + k_2\delta(x-\xi_2), \quad 0 < x < 1, \quad t > 0, \\ u(t,0) = u_L, \quad u(t,1) = u_R, \quad t \geq 0, \\ u(0,x) = \varphi(x), \quad 0 \leq x \leq 1, \end{cases}$$

where $0 < \xi_1 < \xi_2 < 1$ and $\delta(x)$ is a Dirac delta function. We firstly derive the analytical solution of problem (1). Further, we describe the procedure for numerical solution of problem (1) using the standard finite volume method. We provide with numerical illustration for simple test problem.

Keywords: advection-diffusion equation, singular source terms, finite volume method

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