## Lower bound estimation for eigenvalues for many interval BVP's with eigenparameter dependent boundary conditions

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**Abstract:** The present paper deals with multi-interval Sturm-Liouville equations with eigenparameter dependent boundary-transmission conditions. Such type of problems cannot be treated with the usual techniques within the standard framework os classical Sturmian theory. It is well-known that any eigenvalue of the classical Sturm-Liouville problems can be related to its eigenfunction by the Rayleigh quotient and some useful results can be obtained from the Rayleigh quotient without solving the differential equation. For instance, it can be quite useful in estimating the eigenvalues. In this study we present a new technique for investigation some computational aspects of the eigenvalues. Particularly, we give an operator-pencil formulation of the problem and establish lower bound estimation for eigenvalues by using modified Rayleigh quotient.

**Keywords:** Boundary value problems, boundary and transmission conditions, eigenvalues, generalized eigenfunctions, Riesz basis.

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