

# On approximation of first order derivatives of complex-valued functions by finite differences

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**Abstract:** Boundary value problems for partial differential equations involving complex valued functions have important applications in a broad sense ([1–5]). The theory of finite difference method in case of real valued function and its applications to solve boundary value problems for partial differential equations is described in [6]. Complex step method for computing derivatives of real valued functions by introducing a complex step in a strict sense is considered in [7, 8] (see also references therein).

In this presentation, we generalize the well known finite difference method to compute derivatives of real valued function to approximate of complex derivatives  $w_z$  and  $w_{\bar{z}}$  for complex valued function  $w$ . Exploring different combinations of terms, we derive several approximations to compute the first order derivatives of complex valued function  $w$ . The first, second, third and fourth order of accuracy finite differences to calculate derivatives are studied. Error analyses in test examples are carried out by using Matlab program.

**Keywords:** finite difference, complex-valued function, approximation, error.

**2010 Mathematics Subject Classification:** 30E05, 30E10

## REFERENCES

- [1] N.I. Muskhelishvili, Singular Integral Equations, Noordhoff International Publishing, Groningen, 1953.
- [2] I.N. Vekua, Generalized Analytic Functions, Pergamon Press, Oxford, 1962.
- [3] V.N. Monakhov, Boundary-Value Problems with Free Boundaries for Elliptic Systems of Equations (Translations of Mathematical Monographs), AMS, 1983.
- [4] F.D. Gakhov, Boundary Value Problems, Courier Dover Publications, 1990.
- [5] C. Ashyralyev, Numerical algorithms of the solution for singular integral equations and their applications in hydrodynamics, Ylym, Ashgabat, 1994.
- [6] A.A. Samarskii, The theory of difference schemes, Marcell Dekker, Inc., New York, USA, 2001.
- [7] R. Abreu, D.Stich, J. Morales, On the generalization of the Complex Step Method, Journal of Computational and Applied Mathematics, vol. 241, 84–102, 2013.
- [8] R. Abreu, Complex Steps Finite Differences with applications to seismic problems, PhD Tesis, Universidad de Granada, Granada, Spain, 2013.