

Delay Epidemic Model with and without Vaccine

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Abstract: In this work, two models with and without vaccine are constructed. For both model basic reproduction numbers are founded as a

$$R_0^1 = \frac{e^{-\mu\tau} \beta \Lambda}{\gamma + \mu + d}, R_0^2 = \frac{e^{-\mu\tau} \Lambda}{(\gamma + \mu + d)(\mu + r)} \left(\beta + \frac{kr}{\mu + 1 - k} \right).$$

From the basic reproduction ratios it can be seen that when there is no vaccine, disease can only be controlled with reducing the infectious rate β or decreasing the incubation period. However, when there is a vaccine it is enough to increase the rate of the vaccine r to control the disease. In addition to this if the transmission rate of the infectious also decreases then the disease will disappear faster.

Delay effect on this model is very rare. When we are talking about delay on this paper we mean that incubation period. If we have enough vaccine the effect of delay is very tiny. However, if there is no vaccine you can see the effect of delay.

Two equilibria, which are disease-free and endemic equilibria, are found and with using Lyapunov function it is shown that the global stability of each equilibrium for both models. For the first model it is found that DFE E_0 is globally asymptotically stable when $R_0^1 < 1$ and endemic equilibrium E_1 is always asymptotically stable. With using similar method it is shown that E_0 is asymptotically stable when $R_0^2 < 1$ and E_1 is always globally asymptotically stable for model 2. In the last section numerical simulations are given for both models.

Keywords: Delay, Epidemic, Modelling, Vaccine differential equation.

2010 Mathematics Subject Classification: 35N05, 37C75, 34K50

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