A note on stable difference schemes for a third order partial differential equation

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Abstract: Boundary value problems for third order partial differential equations have been a major research area in thermal conductivity [9], micro-scale heat transfer [5] and modern physics. The well-posedness of various nonlocal and local boundary value problems for partial differential and difference equations have been studied extensively by many researchers (see [1–4, 6–9] and the references given therein). In this study, the nonlocal boundary value problem for the third order partial differential equation with a self-adjoint positive definite operator in a Hilbert space is studied. The stability estimates for the solution of the problem are established. In applications, the stability estimates for the solution of two types of third order partial differential equations are obtained. A stable three-step difference schemes for the approximate solution of the problem is presented. The theorem on stability of these difference schemes is established. In applications, the stability estimates for the solution of difference schemes of the approximate solution of nonlocal boundary value problem for third order partial differential equations are obtained. For the numerical analysis, a first and a second order of approximation difference schemes for a one dimensional third order partial differential equation are presented. Moreover, some numerical results are given.

Keywords: Nonlocal boundary value problems, Stability, Difference scheme,Third order partial differential equation, Self-adjoint positive definite operator, Hilbert space.

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