Suitable variations for degenerate modes of the simplex method

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The problem of finding a nontrivial solution of problem

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq 0, \\ \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq 0 \end{cases} \quad (1)$$

at observance of restrictions

$$x_i \geq 0, i = \overline{1, n}. \quad (2)$$

The question of existence of the untrivial solution of this task is key in case of the degenerate modes at the solution of tasks of the linear programming (LP) by simplex method [1]. It is shown in work [2].

Here we will specify an effective method of finding of the solution of an objective if it exists.

Let $A_j = (a_{1j}, \ldots, a_{mj})^T$ be normalized vector. Let us

$$\|A\| = \sqrt{\sum_{i=1}^{m} a_{ij}^2} = 1 \quad (3)$$

It does not reduce a community of consideration of the considered task [2]. Having entered in addition m of variables $x_i \geq 0, i = \overline{n+1, n+m}$ we will transform inequalities (1) to system of the linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + x_{n+1} = 0, \\ \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ + x_{n+m} = 0. \end{cases} \quad (4)$$

The general restriction for all variables now have an appearance

$$x_i \geq 0, \quad i = \overline{1, n+m} \quad (5)$$

Denote by A_{n+k} for any $k = \overline{1, m}$ vector column $m \times 1$ with zero elements in each row expect k^{th} row whode element is one. Then, we can write system

(4) in a vector form

(6) $$\sum_{j=1}^{n+m} x_j A_j = 0$$

We form a set of U in m - measured Euclidean space of R^m with elements $u = (u_1, u_2, \ldots, u_m)^T$ such that

(7) $$u_k = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + x_{n+k}, k = \overline{1, m}$$

representing any linear combinations of vectors $A_j,\ j = \overline{(1, n+m)}$, with the non-negative coefficients of x_j satisfy the condition

(8) $$\sum_{i=1}^{n+m} x_i = 1$$

We will note that the system (4) is uniform; therefore the condition (8) doesn't reduce a community of consideration of a task [2].

Geometrically set of U represents a polyhedron [1] in R^m space, with vertices of $A_j,\ j = \overline{(1, n+m)}$, located on the sphere S with single radiuses. It is known that any point of u limited polyhedron in R^m is representable in the form of a linear combination but at most of $m + 1$ â€“ its vertices

(9) $$u = \sum_{j=1}^{n+m} x_j A_j,$$

at the same time restrictions (5) and (8) are observed. Thus, if

(10) $$0 \in U,$$

that representation takes place $0 = \sum_{j=1}^{n+m} x_j A_j$, so the conditions (5), (6) and (8) are satisfied.

Theorem: In order that the initial task (1),(2) had the decision enough performance of inclusion (10).

This theorem has allowed to make effective algorithm and the program of the solution of LP tasks with the degenerate modes [2]. Made a comparison of the proposed method with known modifications of the simplex method [2].

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References

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