On solution uniqueness of the Cauchy problem for a third-order partial differential equation with time-fractional derivative

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Abstract: Consider the equation

$$\left(\frac{\partial^\sigma}{\partial y^\sigma} - \frac{\partial^3}{\partial x^3}\right) u(x,y) = f(x,y), \tag{1}$$

where $\sigma \in (0,1)$, and $\partial^\sigma/\partial y^\sigma$ stands for the fractional derivative of order σ with respect to y. The fractional differentiation is given by the Dzhrbashyan-Nersesyan operator (see [1]) associated with ordered pair $\{\alpha, \beta\}$, i.e.

$$\frac{\partial^\sigma}{\partial y^\sigma} = D_{0y}^{\{\alpha,\beta\}} = D_{0y}^{\beta-1} D_{0y}^{\alpha}, \qquad \alpha, \beta \in (0,1], \quad \sigma = \alpha + \beta - 1, \tag{2}$$

where $D_{0y}^{\beta-1}$ and D_{0y}^{α} are the Riemann-Liouville fractional integral and derivative, respectively (see [2]).

In paper [3], a fundamental solution of equation (1) and a representation for solution of the Cauchy problem

$$\lim_{y \to 0} D_{0y}^{\alpha-1} u(x,y) = \tau(x), \qquad x \in \mathbb{R}, \tag{3}$$

in the domain $\mathbb{R} \times (0,T)$, were constructed.

Here, we prove a uniqueness theorem for the problem (1), (3) in the class of fast-growing functions satisfying an analogue of the Tychonoff condition.

Keywords: fractional derivative, third-order partial differential equation, Tychonoff's condition, Dzhrbashyan-Nersesyan operator, Cauchy problem

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