Existence and uniqueness of solutions for the first-order nonlinear integro-differential equations with three-point boundary conditions

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Abstract: Mathematical modeling of many processes occurring in the real world often leads to the study of the initial and boundary value problems for ordinary and partial differential equations. Such problems form the basis of mathematical physics. From this point of view integro-differential equations play an important and special role.

In this article, we study existence and uniqueness of solutions of nonlinear differential equations of the type

$$\dot{x}(t) = f(t, x(t)) + \int_0^t k(t, \tau, x(\tau))\, d\tau, \quad t \in (0, T) \tag{1}$$

with three-point boundary conditions

$$Ax(0) + Bx(t_1) + Cx(T) = \alpha. \tag{2}$$

Here A, $B, C \in R^{n\times n}$ are constant square matrices of order n; $\alpha \in R^n$ is a given vector; $\det N \neq 0$; $N = (A + B + C)$,$f : [0, T] \times R^n \to R^n$ and $k : [0, T] \times [0, T] \times R^n \to R^n$ are given functions, and t_1 satisfies the condition of $0 < t_1 < T$. We denote by $C([0, T], R^n)$ the Banach space of all continuous functions from $[0, T]$ into R^n with the norm

$$\|x\| = \max\{|x(t)|, t \in [0, T]\},$$

where $|\cdot|$ is the norm in the space R^n.

We prove new existence and uniqueness results by using the fixed point theorems, and Banach contraction principle.

Similar problems for two-point boundary value problems are considered in [1-3].

Keywords: three-point boundary conditions, integro-differential equations, existence and uniqueness of solutions

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