On the Menger algebras of quasi-open multiplace maps

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Abstract: Investigation of topological spaces via groups, rings, semigroups and other algebraic structures of mappings plays an important role in modern mathematics. Many researchers have focusedtheir efforts on the abstract characterization of topological spaces by semigroups of continuous, open, quasi-open and closed mappings, particularly [2], [3]. Although these deal with the maps of one variable, the wider, but not less important class of multiplace maps are known to have various applications not only mathematics itself, but are also widly used in the theory of many-valued logics, cybernetics and general systems theory. Various natural operations are considered on the sets of multiplace functions. The main operation is the superposition, i.e., the operation which as a result of substitution of some functions into other intsead of their arguments gives a new function.

The study of functions of $n-$ variables was initiated by K. Menger [1] and continued by R.M. Dicker, B.M. Schein, W.A. Dudek and V.S. Trokhimenko. An $(n+1)-$ary operation $[\ ]$ defined on G is superassociative, if for all $x, y_1, ..., y_n, z_1, ..., z_n \in G$ the following identity holds:

$$[[x, y_1, ..., y_n]\, z_1, ..., z_n] = [x, [y_1, z_1, ..., z_n]\, , ..., [y_n, z_1, ..., z_n]]$$

An $(n+1)-$ary algebra $(G; [\ ])$satisfying the above identity is called a Menger algebra of rank n. For $n = 1$ it is an arbitrary semigroup. A function f between topological spaces X and Y is quasi-open if for any non-empty open set $U \subset X$, the interior of $f(U)$ in Y is non-empty. Let $Q(X^n, X)$ denote the Menger algebra of quasi-open functions from X^n into X with composition of functions:

$$[f g_1 ... g_n]\,(a_1, ..., a_n) = f\,(g_1\,(a_1, ..., a_n)\, , ..., g_n\,(a_1, ..., a_n))$$

where $a_1, ..., a_n \in X,\ f, g_1, ..., g_n \in Q(X^n, X)$.

The purpose of this paper is to investigate Menger algebras of quasi-open maps. It is obvious that if X and Y are homeomorphic then the Menger algebras $Q(X^n, X)$ and $Q(Y^n, Y)$ are isomorphic. If $Q(X^n, X)$ and $Q(Y^n, Y)$ are isomorphic, must X and Y be homeomorphic. In general, the answer is no. In this paper, we give an abstract characterization of Menger algebras of quasi-open functions defined on a certain class of topological spaces.

Keywords: Menger algebra, homeomorphism, quasi-open map, lattice equivalence

2010 Mathematics Subject Classification: 20N15, 54C10

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