About the Generalized Dirichlet – Neumann Problem for an Elliptic Equation of High Order

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Abstract: For the elliptic equation $2l-$ th order with constant (and only) real coefficients considered boundary value problem of the job normal derivatives the $(k_j - 1)-$ order, $j = 1, \ldots, l$, where $1 \leq k_1 < \ldots < k_l \leq 2l - 1$. When $k_j = j$ it moves to the Dirichlet problem, and when $k_j = j + 1$ - in the Neumann problem. In this paper, the study is carried out in space $C^{2l,\mu}(\overline{D})$. The sufficient condition of the Fredholm tasks and present a Formula for its index.

Let $s \in [0, 1]$ and $x \in \mathbb{R}^n$, $n \geq 2$. In the work [1], authors show that the following inequality for the fractional Laplacian

$$2f(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^2(x), \tag{1}$$

where $(-\Delta)^s$ is the fractional Laplacian, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

This inequality is using for the maximum principle of the quasi-geostrophic equations. Also, in the works [2] generalized the Cordoba-Cordoba inequality,

$$pf(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^p(x), \tag{2}$$

where $(-\Delta)^s$ is the fractional Laplacian, $p > 0$, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

In the work [3], author generalized these inequalities for the fractional Laplacian. Our main aim of this talk is to establish analogues of the Cordoba-Cordoba inequality and its generalizations for the fractional sub-Laplacian on the homogenous Lie groups.

In this talk, we show an analogue of the Cordoba-Cordoba type inequality for the fractional sub-Laplacian on the homogenous Lie groups. Also, we show generalized analogueof the Cordoba-Cordoba type inequality on the homogenous Lie groups.

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