On a certain class of pseudo-differential operators and related equations

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Abstract: To extend obtained results on Fredholm properties of pseudo-differential operatos on manifolds with non-smooth boundaries [1,2] we start from studying model pseudo-differential operators

$$(Au)(x) = \int\limits_{\mathbb{R}^m} \int\limits_{\mathbb{R}^m} A(x,\xi) e^{i(x-y)\xi} u(y) dy d\xi, \ x \in \mathbb{R}^m,$$

in local Sobolev–Slobodetskii space $H^{s(x)}(\mathbb{R}^m)$ with finite value

$$||u||_{s(x)} \equiv \left(\int\limits_{\mathbb{R}^m} (1+|\xi|)^{2s(x)} |\tilde{u}(\xi)|^2 d\xi \right)^{1/2},$$

where $\tilde{u}$ denotes the Fourier transform of the function u.

Some Fredholm properties for elliptic pseudo-differential operator are obtained for a whole space $\mathbb{R}^m$ and a half-space $\mathbb{R}^m_+$.

Keywords: pseudo-differential operator, variable order, invertibility, Fredholm property

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References

[1] V. B. Vasilyev, Elliptic equations, manifolds with non-smooth boundaries, and boundary value problems, In: New Trends in Analysis and Interdisciplinary Applications. Trends in Mathematics, Research Perspectives. P. Dang et al. (eds.) Birkhäuser, 2017. P. 337-344.

[2] V. B. Vasilyev, Pseudo-differential operators on manifolds with a singular boundary, In: Modern Problems in Applied Analysis. P. Drygas and S. Rogosin (eds.) Birkhäuser, 2018. P. 169-179.