Spectral Properties of the Sturm-Liouville Operator with a Parameter That Changes Sign and Their Usage to the Study of the Spectrum of Differential Operators of Mathematical Physics Belonging to Different Types

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Abstract: This report is devoted to the spectral properties of the Sturm-Liouville operator with a parameter that changes sign and their application to the study of the spectral properties of differential operators of elliptic, hyperbolic and parabolic types.

Consider the differential operator

$$Lu = k\left(y\right)\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + b\left(y\right)u_x + m\left(y\right)u \tag{1}$$

originally defined on $C_0^\infty\left(R^2\right)$, where $k\left(y\right)$ is a piecewise continuous and bounded function changing sign in R, $C_0^\infty\left(R^2\right)$ is the set of infinitely differentiable and compactly supported functions, the coefficients $b\left(y\right), m\left(y\right)$ are continuous functions in R and bounded in each compact set.

It is easily verified that the operator can belong to different types, depending on the sign of the coefficient $k(y)$.

For instance, let $k\left(y\right) = -1$, then the operator L is of elliptic type. If however, $k\left(y\right) = 1$ the operator will be hyperbolic. In the case $k\left(y\right) = 0$, the operator L is of parabolic type.

It is known that the three types of partial differential operators considered here play a special role in mathematical physics. And finally, the type of the operator L may be different at different points if the function $k\left(y\right)$ changes sign in R. We note that such operator is called an operator of mixed type.

Further, let $u\left(x,y\right) \in C_0^\infty\left(R^2\right)$, then it is easy to see that after applying the Fourier transformation with respect to x the operator L takes the form:

$$L_t\hat{u} = -\hat{u}''_{yy} + \left(-k\left(y\right)t^2 + itb\left(y\right) + m\left(y\right)\right)\hat{u}, \tag{2}$$

where $\hat{u}\left(t,y\right) = F_{x\to t}u\left(x,y\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} u\left(x,y\right)e^{-itx}dx$.

Hence it is easy to notice that questions on the existence of the resolvent and spectral properties of the operator L transfer into problems on the existence and spectral properties of the Sturm-Liouville operator with a potential involving a parameter with changing sign.

The following results: 1) the existence of the resolvent, 2) the criterion of the spectrum discreteness, 3) two-sided estimates of approximation numbers (s-numbers) are obtained for operator 2. These results are applied to the study of the spectral properties of differential operators of elliptic, parabolic and hyperbolic types.

Throughout this note we mainly use techniques from our works [1-5].

Keywords: resolvent, spectrum, operator of mixed type, spectral properties, Sturm-Liouville operator with a parameter

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