About Volterra's nonlocal boundary value problem with displacement for the wave equation

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Abstract: Recently the interest in boundary value problems of the wave equation sharply have increased due to the growing needs of Geophysics and Seismology. In this paper we show that the number of boundary value problems for the wave equation, taken inside the characteristic quadrangle are Volterra. The main results of this work are presented by following theorems:

Theorem 1. *If $\alpha^2 - \beta^2 \neq 0$, then a nonlocal boundary value problem with displacement*

$$Lu = u_{xy}(x, y) = f(x, y), (x, y) \in \Omega; \tag{1}$$

$$\alpha \cdot u(0, y) + \beta \cdot u(1, 1 - y) = 0, \tag{2}$$

$$\alpha \cdot u(x, 0) + \beta \cdot u(1 - x, 1) = 0, \tag{3}$$

is strongly solvable, and the inverse operator L^{-1} to the strong operator of problem (1)–(3) is completely continuous and Volterra.

Theorem 2. *If $\alpha^2 - \beta^2 \neq 0$, then nonlocal boundary value problem*

$$Lu(x, y) = u_{xy}(x, y) = f(x, y),\ \ f(x, y) \in L^2(\Omega), \tag{4}$$

$$\alpha u(0, y) + \beta u(y, 0) = 0, \tag{5}$$

$$\alpha u(x, 1) + \beta u(1, x) = 0 \tag{6}$$

is strongly solvable in space $L^2(\Omega)$, and the operator L^{-1} inverse to the strong operator of problem (4)–(6) is completely continuous and Volterra.

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