# On the Spectral Properties of a Wave Operator Perturbed by a Lower-Order Term

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**Abstract:** The incorrectness of the minimal wave operator is well known, since zero is an infinite-to-one eigenvalue for it. As our study showed, the situation changes if the operator is perturbed by a low-order term containing the spectral parameter as a coefficient, and eventually the problem takes the form of a bundle of operators. The resulting bundle of operators is easily factorized by first-order functional-differential operators whose spectral properties are easily studied by the classical method of separation of variables. Direct application of the method of separation of variables to the original bundle of operators encounters insurmountable difficulties. In this work we consider the following spectral problem:

$$u_{xx} - u_{yy} = -2\lambda u_x + \lambda^2 u, \tag{1}$$

$$u|_{y=0} = 0, \ u|_{x=0} = \alpha\, u|_{x=1}\,, \ |\alpha| = 1. \tag{2}$$

We investigated the spectral properties of task(1)-(2) and obtain the eigenvalues and corresponding eigenfunctions, which form an orthonormal basis of the space $L^2(\Omega)$. The spectrum of the beam of operators (1)-(2) consists of two series of infinite eigenvalues.

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## References

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