Compactness Of The Commutators Of Riesz Potential On Global Morrey-type Space.

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Abstract: In this paper we obtain the sufficient conditions of compactness of the commutator for the Riesz potential $[b, I_\alpha]$ in global Morrey-type spaces $GM^w_{p\theta}$.

Let $1 \leq p \leq \infty$, w be a measurable non-negative function on $(0, \infty)$. The Global Morrey-type space $GM^w_{p\theta} \equiv GM^w_{p\theta}(\mathbb{R}^n)$ is defined as the set of all functions $f \in L^{loc}_p(\mathbb{R}^n)$ with finite quasi-norm $\|f\|_{GM^w_{p\theta}} \equiv \sup\limits_{x\in\mathbb{R}^n} \left\| w(r) \|f\|_{L_p(B(x,r))} \right\|_{L_\theta(0,\infty)}$, where $B(t, r)$ the ball with center at the point t and of radius r.

For the function $b \in L_{loc}(\mathbb{R}^n)$ we denote by M_b the multiplication operator $M_b f = bf$, where f is a measurable function. Then the commutator for the Riesz potential I_α and the operator M_b is defined by the equality $[b, I_\alpha] = M_b I_\alpha - I_\alpha M_b = C_{n,\alpha} \int\limits_{\mathbb{R}^n} \frac{[b(x)-b(y)]f(y)}{|x-y|^{n-\alpha}} dy$. It is said that the function $b(x) \in L_\infty(\mathbb{R}^n)$ belongs to the space $BMO(\mathbb{R}^n)$ if $\|b\|_* = \sup_{Q\subset\mathbb{R}^n} \frac{1}{|Q|} \int\limits_Q |b(x) - b_Q| \, dx = \sup_{Q\in\mathbb{R}^n} M(b, Q) < \infty$, where Q - cub $\mathbb{R}^n$ and $b_Q = \frac{1}{|Q|} \int\limits_{\mathbb{R}^n} f(y)dy$.

We denote by $VMO(\mathbb{R}^n)$ the BMO -closure of the space $C^\infty_0(\mathbb{R}^n)$, where $C^\infty_0(\mathbb{R}^n)$ the set of all functions in $C^\infty(\mathbb{R}^n)$ with compact support.

Theorem 2. Let $0 < \alpha < n(1 - \frac{1}{q})$, $1 \leq p_1 < p_2 < \infty$, $\alpha = n(\frac{1}{p_1} - \frac{1}{p_2})$, or $1 \leq \ p_1 < \infty$, $1 \leq p_2 < \infty$ and $n(\frac{1}{p_1} - \frac{1}{p_2}) < \alpha < \frac{n}{p_1}$, $\alpha < n(1 - \frac{1}{p_2})$, $0 < \theta_1 \leq \theta_2 \leq \infty$, $\theta_1 \leq 1$, $w_1 \in \Omega_{p_1\theta_1}$, $w_2 \in \Omega_{p_2\theta_2}$, and let $\left\| w_2(r) \frac{r^{\frac{1}{p_2}}}{(t+r)^{\frac{n}{p_1}-\alpha}} \right\|_{L_{\theta_2(0,\infty)}} \leq c \|w_1(r)\|_{L_{\theta_1(t,\infty)}}$ for all $t > 0$, where $c > 0$ not depend on t, and $b \in VMO(\mathbb{R}^n)$. In addition, let the commutator $[b, I_\alpha]$ bounded from $GM^{w_1}_{p_1\theta_1}$ to $GM^{w_2}_{p_2\theta_2}$. More over, let $\|[b, I_\alpha]\|_{GM^{w_2}_{p_2\theta_2}} \leq \|b\|_* \|f\|_{GM^{w_1}_{p_1\theta_1}}$ Then the commutator $[b, I_\alpha]$ is a compact operator from $GM^{w_1}_{p_1\theta_1}$ to $GM^{w_2}_{p_2\theta_2}$

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