Degenerate operators for ordinary differential equations

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Abstract: We consider the spectral problem for operators generated by ordinary differential equations. A degenerate operator means an operator whose point spectrum fills the whole complex plane, or a resolvent set fills the whole complex plane.

Conditions are found for the coefficients of the general differential equation for which the corresponding operator is degenerate.

Theorem. *Let the operator L generated in $L_2(0,1)$ by the differential expression $l(y) = y^{(n)} + \sum_{j=1}^{n} p_j(x) y^{(n-j)}$, and the boundary conditions $U_j(y) = y^{(j-1)}(0) - \alpha \cdot \alpha_2^{(j-1)} y^{(j-1)}(1) = 0$, $j = 1, 2, \ldots, n$, where $\alpha_k = e^{i\frac{2\pi(k-1)}{n}}$, $k = 1, 2, \ldots, n$, being the n-th roots of unity with $\alpha_1 = 1$. If the condition*

$$p_k(x) = \alpha_2^k p_k(1 + \alpha_2 x), \quad k = 1, 2, \ldots, n, \ 0 \leq x \leq 1,$$

is satisfied, then the operator L is degenerate, that is, the characteristic determinant $\Delta_L(\rho) \equiv const$. Furthermore, $\Delta_L(\rho) = 1 - \alpha^n$, and if $\alpha^n \neq 1$, then L^{-1} is a Volterra operator; if $\alpha^n = 1$, then $\rho(L) = \varnothing$ and $\sigma(L) = \mathbb{C}$.

Keywords: ordinary differential equations, degenerate operator, spectrum, resolvent set

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