Generalized three-dimensional singular integral equation by tube domain

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Abstract: In this work, we investigate one class of three- dimensional complex integral equation by tube domains, ave in lower basis and lateral surface and may have singularity.

Let Ω denote the tube domain $\Omega = \{(z,t) : a < t < b, |z| < R\}$. Lower ground this cylinder denote by $D = \{t = a, |z| < R\}$ and lateral surface denote by $S = \{a < t < b, |z| = R\}, z = x + iy$. In Ω we shall consider the following integral equation

$$\varphi(t,z) + \int_a^t \frac{K_1(t,\tau)}{\tau - a}\varphi(\tau,z)d\tau + \frac{1}{\pi}\iint\limits_D \frac{\exp[i\theta]K_2(r,\rho)}{(R-\rho)(s-z)}\varphi(t,s)ds$$

$$+\frac{1}{\pi}\int_a^t \frac{d\tau}{\tau - a}\iint\limits_D \frac{K_3(t,\tau;r,\rho)}{(R-\rho)(s-z)}\exp[i\theta]\varphi(\tau,s)ds = f(t,z), \qquad (1)$$

where $\theta = \arg s, s = \xi + i\eta, ds = d\xi d\eta, \rho^2 = \xi^2 + \eta^2, r^2 = x^2 + y^2$, $K_1(t,\tau) = \sum\limits_{j=1}^{n} A_j \ln^{j-1}(\frac{t-a}{\tau-a})$, $K_2(r,\rho) = \sum\limits_{l=1}^{m} B_l \ln^{l-1}(\frac{R-r}{R-\rho})$, $K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$, $A_j(1 \le j \le n), B_l(1 \le l \le m)$ are given constants, $f(t,z)$ are given function, $\varphi(t,z)$ unknown function. In depend from the roots of the characteristics equations

$$\lambda^n + \sum_{j=1}^{n} A_j(j-1)!\lambda^{n-j} = 0, \mu^m + \sum_{l=1}^{m} B_l(l-1)!\mu^{m-j} = 0$$

obtained representation the manifold solution of the integral equation (1), by m arbitrary functions $\Phi_l(t,z)(1 \le l \le m)$ analytically by variables z and continuously by variables t and n arbitrary function $C_j(z)$ $(1 \le j \le n)$ continuously by variables z.

Keywords: tube domain; singular kernels; manifold solution; logarithmic singularity.

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