On a boundary condition of Bitsadze-Samarskii for the Lavrent'ev-Bitsadze equation

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Abstract: This report is devoted to study the Bitsadze-Samarskii boundary condition for the Lavrent'ev-Bitsadze equation. In the domain D we consider the two-dimensional integral potential:

$$u = L_B^{-1} f = \begin{cases} \Delta_N^{-1} f^{-}(x_1, x_2), \ (x_1, x_2) \in D^{-}, \\ \Box^{-1} f^{+}(x_1, x_2) + \Box_0^{-1} \tau(x_1, x_2) + \Box_1^{-1} \nu(x_1, x_2), \ (x_1, x_2) \in D^{+}, \end{cases}$$

where

$$\Delta_N^{-1} f^{-}(x_1, x_2) = \int_{D^{-}} \varepsilon_2^{-}(x_1 - \xi_1, x_2 - \xi_2) f^{-}(\xi_1, \xi_2) d\xi_1 d\xi_2 \tag{1}$$

the two-dimensional Newton potential and the wave potential:

$$\Box^{-1} f^{+}(x_1, x_2) = \int_0^b d\xi_2 \int_0^1 \varepsilon_2^{+}(x_1 - \xi_1, x_2 - \xi_2) f^{+}(\xi_1, \xi_2) d\xi_1 \tag{2}$$

and surface wave potentials:

$$u_\tau(x_1, x_2) = \Box_0^{-1} \tau(x_1, x_2) = \varepsilon_2^{+} * \tau(x_1) \delta^{(/)}(x_2), \tag{3}$$

$$u_\nu(x_1, x_2) = \Box_1^{-1} \nu(x_1, x_2) = \varepsilon_2^{+} * \nu(x_1) \delta(x_2). \tag{4}$$

When $f(x_1, x_2) \in C^{1+\alpha}(\overline{D})$ the integral operator $L_B^{-1} f(x_1, x_2) \in C^{1+\alpha}(\overline{D}) \cap C^{2+\alpha}(\overline{D^{-}}) \cap C^{2+\alpha}(\overline{D^{+}})$ satisfies the equation of mixed (elliptic-hyperbolic) type:

$$L_B u(x_1, x_2) = \mathrm{sgn} x_2 u_{x_1 x_1}(x_1, x_2) - u_{x_2 x_2}(x_1, x_2) = f(x_1, x_2), \ (x_1, x_2) \in D, \tag{5}$$

which is called the Lavrent'ev-Bitsadze equation. In this paper, by using techniques from our works [1,2] we found the boundary conditions for the Bitsadze integral operator $L_B^{-1} f$.

Keywords: Newton potential, wave potential, surface wave potentials, Bitsadze-Samarskii boundary condition, Lavrent'ev-Bitsadze equation

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References

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