On the unique solvability of a nonlocal problem with integral condition for the impulsive partial differential equation third order

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Abstract: The nonlocal problem with integral condition for the impulsive partial differential equation third order is investigated. Conditions of the unique solvability to nonlocal problem are established in the terms of initial data.

In this report, on $\Omega = [0,T] \times [0,\omega]$ the nonlocal problem with integral condition for the impulsive partial differential equation is considered

$$u_{txx} = A(t,x)u_{xx} + B(t,x)u_{tx} + C(t,x)u_x + f(t,x), \qquad t \neq t_i, \tag{1}$$

$$P_i(x)u(t_i+0,x) - S_i(x)u(t_i-0,x) = \varphi_i(x), \qquad i = \overline{1,m}, \quad x \in [0,\omega], \tag{2}$$

$$L(x)u_{xx}(0,x) + \sum_{j=1}^{k} \int_{t_{j-1}}^{t_j} M_j(t,x)u_{xx}(t,x)dt = \varphi_0(x), \quad x \in [0,\omega], \tag{3}$$

$$u(t,0) = \psi_1(t), \qquad u_x(t,x)\big|_{x=0} = \psi_2(t), \qquad t \in [0,T], \tag{4}$$

where $u(t,x)$ is unknown function, the functions $A(t,x)$, $B(t,x)$, $C(t,x)$, and $f(t,x)$ are continuous on Ω, the functions $P_i(x)$, $S_i(x)$, $\varphi_i(x)$ are twice continuously differentiable on $[0,\omega]$, $i = \overline{1,m}$, $0 < t_1 < t_2 < ... < t_m \leq T$, the functions $L(x)$, $\varphi_0(x)$, $M_j(t,x)$ are continuous on $[0,\omega]$, Ω, respectively, $j = \overline{1,k}$, the functions $\psi_1(t)$ and $\psi_2(t)$ are continuously differentiable on $[0,T]$.

Problem (1)–(4) is reduced to equivalent problem, consisting of nonlocal problem with integral condition for impulsive hyperbolic equation second order and functional relations. Algorithm of finding approximate solution to researching problem is constructed and the its convergence is proved. The conditions of classical solvability to problem (1)–(4) are established by method of introduction additional functional parameters [1, 2].

Keywords: partial differential equation, nonlocal problem, impulse effect

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